

ISSN: 2329-8243 (Print)

ISSN: 2329-8235 (Online)

FIN GEOMETRY OPTIMIZATION OF NON-NEWTONIAN FLUID, FLOWING THROUGH AN ANNULUS PIPE, USING ENTROPY GENERATION MINIMIZATION METHOD

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ARTICLE DETAILS

Article History:

Received 7 November 2017

Accepted 10 December 2017

Available online 5 January 2018

ABSTRACT

In the present study a non-Newtonian fluid flowing in an annulus pipe, has been simulated numerically. The considered geometry is an annulus pipe equipped with some fins. Heat transfer and entropy generation for some different fin geometries have been characterized through a written computer program. The written code is based on finite element method using Hermitian weight functions. Using first order Hermitian weight function, keeps all derivations continuous in the integrating domain, while improving convergence rate and also the accuracy of the solution. The geometry of fins is optimized using "Entropy Generation Minimization", in order to reach maximum heat transfer rate. Increasing length of a fin, improves heat transfer while increasing pressure drop of the flow which in turn decrease heat transfer; therefore the optimizing method should simultaneously consider the two mentioned effects. In the present work, simulation of the non-Newtonian fluid has been accomplished using Power Law for modeling of non-Newtonian flow, while continuity, mass and energy reservation laws have been considered. Entropy generation rate is then obtained using second law of thermodynamics. Different fin geometries and arrangements are compared in order to find the optimum condition.

KEYWORDS

Annulus flow, FEM (Finite Element Method), hermitian functions, fin, optimization, generated entropy, Entropy Generation Minimization (EGM)

1. INTRODUCTION

Heat transfer processes play an important role in many industrial applications. The fluids which are used in most industries are viscous and non-Newtonian. Taking the assumption of ideal fluid to simulate these types of fluids causes ineligible errors. Having non linear shear stress and shear rate, is the most prominent characteristic of non-Newtonian fluids. These fluids can be classified into semi plastic or elastic groups depending on their chemical structure. Nowadays, annulus flows in concentric pipes are applied in many industries specifically in heating or cooling applications. In some conditions that we are to increase the heat transfer rate, using fins, particularly spiral fins, is a custom approach. All types of fins increase heat transfer besides increasing pressure drop of the flow, although the first effect is desired the second effect (increasing pressure drop) should be minimized. This issue makes the selection of fins geometry dominantly important. In the present study we are going to find the optimum geometry of fins by EGM method. It should be noted that thermal resistance and pressure drop effects are simultaneously simulated in thermal interactions by EGM method. To reach the mentioned aim, the flow domain should be simulated at the beginning step. Since the shear stress in non-Newtonian fluids is not a linear function of velocity, the governing equations should be first linearized, by applying some linearization methods. One of the hardships of applying linearizing methods, is providing the solution convergence. This issue is due to existence of two non-linear terms in N-S equation. It is known that the total entropy generation is because of heat transfer and viscous friction effects. Increasing the length of thermal fins leads to increment of heat transfer rate and also the viscous friction. In the present work we are going to find the optimum fin geometry by considering both friction and heat transfer effects. The geometric parameters which are going to be optimized are width and length of fins and also the distance between them. The selected length scale for this study is the hydraulic diameter of the pipe.

Many researches have been devoted to simulation of the fluid flow in annulus pipes both those equipped with fins and those not equipped. In 1995, entropy generation minimization in thermal systems has been investigated by a researcher [1]. He demonstrated that entropy generation minimization improves the performance of the system. A group of researchers has analyzed the second law of thermodynamics in an annulus pipe [2]. Considering volume entropy generation rate, they perceive that there is entropy increment in some parts of the system depends on their performance. Irreversibility of the flow in a rotating annulus pipe has been characterized by a scholars in 2002 [3]. The magnitude of generated entropy has been calculated in both constant temperature and constant pressure conditions in the mentioned study. A group of young researchers in 1992, investigated the heat transfer rate for a non-Newtonian fluid flowing in an annulus pipe [4]. Forced and free convective heat transfers have been formulated for fully developed flows in annulus pipes, by another researchers [5]. A numerical simulation of Bingham fluids flowing in vertical eccentric annulus pipes has been accomplished [6]. The behavior of elastic fluids flow in annulus pipe has been investigated [7]. A researcher in 1984, theoretically obtained the optimum geometry of the fin for external Newtonian flow with forced convective heat transfer [8]. At the beginning, they derived an equation for computing entropy generation rate in the flow; then by minimizing the mentioned equation the optimum geometric parameters were obtained. The present study is devoted to improvement of heat transfer in a non-Newtonian flow in an annulus pipe, by applying optimal fins.

2. NON-NEWTONIAN FLUIDS

A simple criterion for recognition of non-Newtonian fluids is the $\frac{\tau}{\dot{\gamma}}$ ratio. Where, τ is the flow shear stress and $\dot{\gamma}$ is strain rate. There is a consensus among researchers that if the mentioned ratio is not a

constant value, the fluid behaves as a non-Newtonian fluid. In other word, non-Newtonian fluids consist of some molecules with different size, shape and adhesion; these issues make the

ratio τ not remain constant. Non-Newtonian fluids are classified into time γ dependent fluids and time independent ones. Time- dependent non-Newtonian fluids are those that have viscosity changing by time in a constant strain rate. And time-independent non-Newtonian fluids are those with constant viscosity in a constant strain rate. Many rheological models have been introduced till now. The main objective of these models is to find a relation between shear stress and fluid velocity gradient. Two popular models have been mentioned below:

2.1 Bingham plastic model

This model uses two time-independent parameters to introduce the correlation between stress and velocity. (Equation (1)) this relation consists of two terms, the first one is residual stress of the fluid and the second one is a linear function of γ .

$$\tau = \tau_0 + \mu_p \gamma \quad (1)$$

Where τ_0 accounts for yield stress, μ_p denotes plastic viscosity.

2.2 Power Law model

Same as the previous model, this model consists of two time-dependent parameters. In Bingham model there is a linear relation between stress and strain, but power law model introduces a nonlinear relation which is more accurate for thin boundary layer problems. The suggested relation is:

$$\tau = k\gamma^n \quad (2)$$

Where, both k and n depend on fluid temperature and property. k, is the stability factor and the viscosity of the fluid is increased by increment of k value. n, demonstrates the degree of non-Newtonian behavior of the fluid. For instance, n=1, simulates the Newtonian behavior, 0<n<1, simulates the semi-plastic and shear-thinning behavior and n>1, associates with shear-thickening behavior. Variation of shear stress with strain rate is shown in Figure 1.

3. HEAT TRANSFER IN ANNULUS PIPES

Heat transfer in annulus pipes is a common process in industrial manufacturing of most fluids. Double pipe heat exchangers are dominantly involved with the non-Newtonian fluid heat transfer phenomena. Geometrical symmetry and behavior of the non-Newtonian fluid are the two parameters which mainly affect the flow and temperature distribution. Since most of fluids in experiment, behave in non-Newtonian manner, simulation of the fluid flow and heat transfer in an annulus pipe is a custom approach to control many processes in industry specially food industry. In the present study a fully developed non-Newtonian flow is numerically simulated using power law model and considering rheological effects of the fluid. The considered conditions are similar to the real condition of thermal process in heat exchangers.

4. NUMERICAL SIMULATION OF FLOW

Here in this study a numerical code is written in order to simulate the flow distribution and heat transfer. The first stage of modeling the non-Newtonian flow in annulus pipe is to discretize the flow domain in to some elements. FEM discretizing method is used in the present code. The flow domain is fully meshed by triangular elements using the "Delani" method. This method is completely mentioned in reference [9]. Figure 2 demonstrates a typical meshing, for the fin equipped annulus pipe. (The symmetry of computational domain permits the restriction of solution to only half of the pipe).

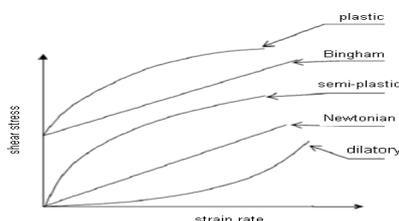


Figure1: Shear/strain variation for Newtonian and non-Newtonian fluids

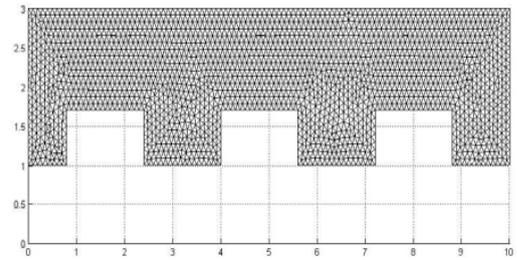


Figure 2: Schematic of fin equipped annulus pipe meshing

In order to discretize the equations with FEM method, all equations should be converted into integral form.

$$\begin{aligned} L(\phi) &= g \\ I(\phi) &= \int_s F(\bar{r}, \phi, \dot{\phi}, \dots) \end{aligned} \quad (3)$$

In the above equations, L is the operator and g is the source term. It should be noted that the two above equations are equal and the only difference is that, one is in differential form and the other in integral form [10]. The second equation can be linearized by the following method:

$$\begin{aligned} \phi &= \sum_{n=1}^{\infty} a_n u_n + u_0 \\ I(\phi) &= I(a_0 + a_1 + \dots) \end{aligned} \quad (4)$$

as, are obtained by using "Raily Risher" or "weighted residual" method.

$$\frac{\partial(I)}{\partial(a_1)} = 0, \dots \quad (5)$$

$$\begin{aligned} L(\phi) &= g, \\ R &= L(\phi - \tilde{\phi}) \\ L(\tilde{\phi}) - g &= \int f_i(x) R dv \\ \bar{\phi} &= \sum_{n=1}^{\infty} a_n u_n \Leftrightarrow < f_i(x), R > = 0 \\ \sum a_n < f_i(x), L(u_n) > &= < f_i(x), R > \end{aligned} \quad (6)$$

In the above equations, ϕ is the exact solution, $\tilde{\phi}$ is the estimated solution and f_i is the weight function. In the last proposed method, choosing different weight function leads to formation of different methods. "Galerkin" method is one of these methods. In Galerkin method, weight function is equal to shape function m , and thw e following relation is governed:

$$\int w_m R dv = 0 \quad m = 1, 2, \dots \quad (7)$$

In this part, the consequences of deriving finite element equations are described. At the first stage, the geometry is divided into "E" finite elements. Each unknown variable of the element "e" is supposed as below: (eg. Velocity)

$$\bar{U} = \begin{Bmatrix} u(x, y, z) \\ v(x, y, z) \\ w(x, y, z) \end{Bmatrix} = [N] \bar{Q}^{(e)} \quad (8)$$

In the above relation (e), isQ the vector of freedom degrees of element nodal displacement, and [N] is the matrix of shape functions. Weighted residual integration over an element is assumed zero in Galerkin method. In this method N_i , is the interpolating weight function. The function, I, is defined as summation of E quantities of elements, I(e):

$$I = \sum_{e=1}^E I^{(e)} \quad (9)$$

Writing equations in matrix for, leads to:

$$\text{Where,} \quad [K] Q = P \quad (10)$$

$$[K \sim] \sum_{e=1}^E [K^{(e)}] \quad (11)$$

is the total stiffness matrix and

$$\vec{P} = \vec{P}_C + \sum_{e=1}^E \vec{P}_1^{(e)} + \sum_{e=1}^E \vec{P}_s^{(e)} + \sum_{e=1}^E \vec{P}_b^{(e)} \quad (12)$$

is the matrix of total force. At the next step, boundary and initial conditions are applied to modify matrices p and k; then the mentioned Equation (10), is solved using matrix solution methods. In finite element method, all variables are defined at the nodes of elements and magnitude of a variable at any point in the element is described as a function of nodal variables and coordinates of the point. This function can be considered as interpolating function.

5. FUNCTIONS OF INTERPOLATION

One of the prominent effective agents on the precision of solution is the type of chosen weight or shape functions. Converging the solution of the problem, is an important issue that is hard to reach in most cases. In the present study, Hermitian weight functions are used to facilitate the convergence of solution by preserving the derivatives continuous. Any function, $\phi(x)$ which is shown in Figure 3, can be estimated using Hermitian functions:

$$\begin{aligned} \phi(x) &= \sum_{i=0}^2 \sum_{k=0}^N H_{ki}^{(N)}(x) \phi_i^{(k)} \\ &= \sum_{i=0}^2 [H_{0i}^{(N)}(x) \phi_i^{(0)} + H_{1i}^{(N)}(x) \phi_i^{(1)} + H_{2i}^{(N)}(x) \phi_i^{(2)} + \dots + H_{Ni}^{(N)}(x) \phi_i^{(N)}] \end{aligned} \quad (13)$$

Where, $\phi_i^{(k)}$ are unknown parameters.

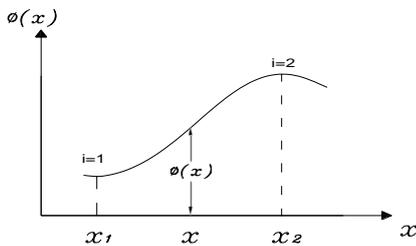


Figure 3: A typical one-dimensional function that is interpolated through x_1, x_2

Hermitian polynomials have the following characteristic:

$$\begin{cases} \frac{d^2 H_{ki}^{(N)}}{dx^r}(x_p) = \delta_{ip} \delta_{kr} \\ i, p = 1, 2, \dots \\ k, r = 0, 1, 2, \dots, N \end{cases} \quad (14)$$

Where, x_p is the value of x at the p th point and δ_{ip} , is kroncker delta. The first order Hermitian weight function for an element is written as below:

$$\begin{aligned} H_{01}^{(1)}(x) &= \frac{1}{3}(2x^3 - 3/x^2 + 1^3) \\ H_{02}^{(1)}(x) &= -\frac{1}{3}(2x^3 - 3/x^2) \\ H_{11}^{(1)}(x) &= \frac{1}{2}(x^3 - 2/x^2 + 1^2)x \\ H_{12}^{(1)}(x) &= \frac{1}{2}(x^3 - 1x^2) \end{aligned} \quad (15)$$

Each part of the above relation has a special concept which is shown in Figure 4.

6. SIMULATION OF NON-NEWTONIAN FLUID

The fluid which is considered in this study is a semi-plastic calculated using power law relation:

$$\begin{aligned} \tau &= 16.79 + K(T) [\dot{\gamma}]^{n(T)} \\ n(T) &= a_n \exp(b_n \cdot T) \\ K(T) &= a_k \exp(-b_k \cdot T) \end{aligned} \quad (16)$$

Momentum equation for power law model fluid at "z" and "r" directions in cylindrical coordinates are respectively as below:

$$\rho \left(v \frac{\partial u}{\partial r} + u \frac{\partial v}{\partial z} \right) = -\frac{\partial p}{\partial z} + \mu_a \left(\frac{1}{r} \frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial r^2} \right) + 2 \frac{\partial \mu_a}{\partial z} \frac{\partial u}{\partial z} + \frac{\partial \mu_a}{\partial r} \left(\frac{\partial u}{\partial r} + \frac{\partial v}{\partial z} \right) \quad (17)$$

$$\rho \left(v \frac{\partial v}{\partial r} + u \frac{\partial v}{\partial z} \right) = -\frac{\partial p}{\partial r} + \mu_a \left(\frac{1}{r} \frac{\partial v}{\partial r} + \frac{\partial^2 v}{\partial r^2} - \frac{v}{r^2} \right) + 2 \frac{\partial \mu_a}{\partial r} \frac{\partial v}{\partial r} + \frac{\partial \mu_a}{\partial z} \left(\frac{\partial u}{\partial r} + \frac{\partial v}{\partial z} \right) \quad (18)$$

Energy equation is also written:

$$\rho C_p \left(v \frac{\partial T}{\partial r} + u \frac{\partial T}{\partial z} \right) = \lambda \left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} \right) + \mu \phi \quad (19)$$

Where,

$$\phi = 2 \left[\left(\frac{\partial v}{\partial r} \right)^2 + \left(\frac{v}{r} \right)^2 + \left(\frac{\partial u}{\partial z} \right)^2 \right] + \left(\frac{\partial u}{\partial r} + \frac{\partial v}{\partial z} \right)^2 \quad (20)$$

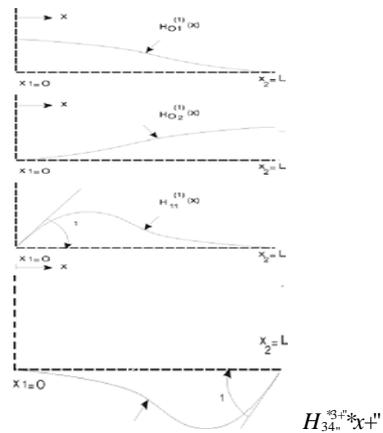


Figure 4: Variation of Hermitian first order polynomial between two points

7. CALCULATION OF ENTROPY GENERATION

Second law of thermodynamics is a useful concept non-Newtonian fluid. The fluid viscosity is which is dominantly used in optimization of engineering problems. Increasing heat transfer of heat exchangers or decreasing the energy consumption of pumps are the two examples in which second law of thermodynamics plays an important role. The process of optimization in most problems confined to entropy generation calculation and minimization. In the present study, the generated entropy can be obtained by Equation (21),

$$\begin{aligned} S_{gen}^m &= \frac{k}{T^2} \left[\left(\frac{\partial T}{\partial r} + \frac{\partial T}{\partial z} \right)^2 + \right. \\ &\left. \frac{\mu}{T} \left\{ 2 \left[\left(\frac{\partial v}{\partial r} \right)^2 + \left(\frac{v}{r} \right)^2 + \left(\frac{\partial u}{\partial z} \right)^2 + \frac{1}{2} \left(\frac{\partial v}{\partial z} + \frac{\partial u}{\partial r} \right)^2 \right] \right\} \right] \geq 0 \end{aligned} \quad (21)$$

Since stress is a function of temperature, the energy, momentum and power law equations should be solved simultaneously. The algorithm of computer program which is written in this work is shown in Figure 5. This flowchart demonstrates the procedure of computer program.

8. VERIFICATION OF THE COMPUTER PROGRAM

As previously mentioned, a computer program in which a new modeling approach is applied, has been developed to characterize effects of fin geometry on the heat transfer performance in non-Newtonian fluids flow. Before discussing the results of our computer program, it is essential to assure correctness of the program performance.

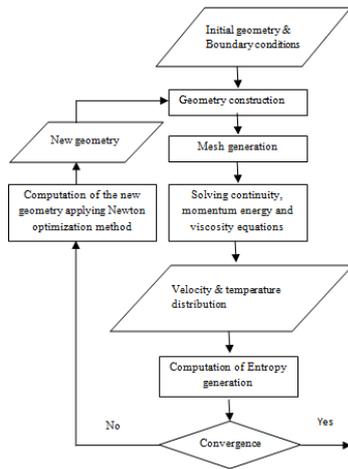


Figure 5: The computer program flowchart

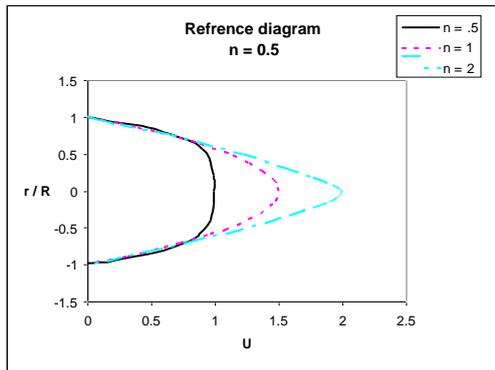


Figure 6: Results of ref [5, 6] for n=0.5, n=1 and n=2

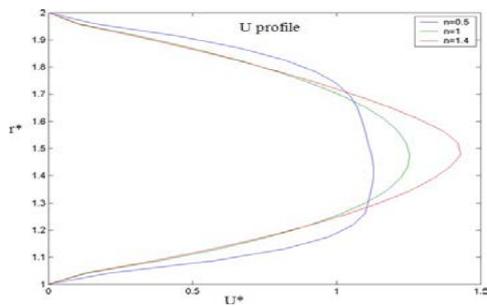


Figure 7: Results of ref [11, 12] for n=0.5, n=1 and n=2

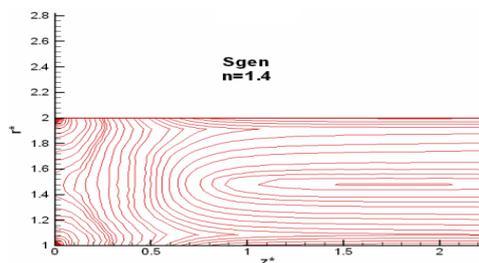


Figure 8: Entropy generation countours for non-Newtonian fluids of n=1.4 by applying temperature constant boundary condition

To achieve this objective, the results of the computer program, have been compared with the results of reference, for different powers allocated to the power law model [11,12]. The aforesaid comparison demonstrated a good agreement between the results of our model and diagrams. Figure 6 accounts for reports of other researches for n=0.5, n=1, n=2. Figure 7 also demonstrates some other results for n=0.5, n=1, n=1.4. Taking these curves into consideration, it is concluded that, increasing in value of n, results in increment of maximum axial velocity. A typical generated entropy contour of non-Newtonian fluid flowing in an annulus pipe with n=1.4 is shown in Figure 8.

9. OPTIMIZATION PROCEDURE OF THE PROBLEM

Developing the computer program and modeling procedure were described in previous sections. It was mentioned that the computer program uses geometrical characteristics of fins and annulus pipe as input, to characterize the non-Newtonian fluid flow and develop generated entropy, velocity and temperature distribution. At this step, the optimum geometry which leads to maximum heat transfer and minimum pressure drop should be found. Fin geometry is shown in Figure 9. Because of the pipe symmetry just half of the annulus pipe is demonstrated in the figure. Variables of the optimization problem are the fin width, a, the fin height, b, and the distance between two successive fins, c. Optimizing methods are very similar to try and error methods. In both methods, design parameters are modified by the results of the developed model at each step. In optimization procedure, design modification is accomplished by means of a defined objective function and the designer interferes in the problem procedure just by choosing the suitable form for this function. At the present section of this study, an objective function, $F(\Phi)$, is defined where Φ , is the vector formed by design variables. The aim of the optimization algorithms is to find Φ^* so that the following relation is satisfied: $F(\Phi^*) = \min(F(\Phi))$ (22)

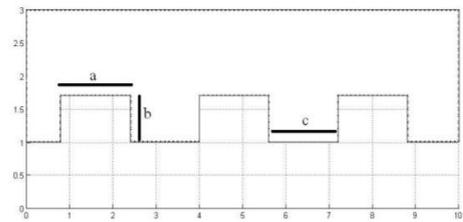


Figure 9: Parameters to be optimized in the solution procedure (a, b, c)

Table1: Solution of optimization problem- optimized characteristics of fins for the non-newtonian fluid flowing in an annulus pipe

	a	b	c
n=1	1.40	1.41	1.29
n=0.5	1.30	1.32	1.22
n=1.4	1.46	1.46	1.34

There is a consensus that among a variety of methods, step descent method is particularly promising due to its high efficiency. The variables are modified due to the following relation at each step: $\Phi_{k+1} = \Phi_k + p_k \alpha_k$ (23)

Where p_k , and α_k account for the direction of variation and variation step respectively. The following relation is developed for selecting α_k : $\alpha_k = \alpha_0 / k^a$ (24)

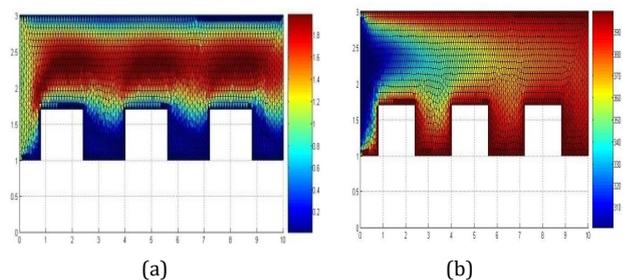


Figure 10: a) Velocity distribution of the flow in the annulus pipe; b) Temperature contours in the upper half of the annulus pipe

If Φ^* is the minimum value of the function and the value of the function at each step is shown by Φ_k and c_k is the connecting vector of these values, ($\Phi^* = \Phi_k + c_k$), Taylor expansion will be formed as below:

$$F(\Phi^*) = F(\Phi_k + c_k) \approx F(\Phi_k) + c_k \cdot g(\Phi_k) \quad (25)$$

Where,

$$g(\Phi) = \left[\frac{\partial F(\Phi)}{\partial \Phi_1} \quad \frac{\partial F(\Phi)}{\partial \Phi_2} \quad \dots \quad \frac{\partial F(\Phi)}{\partial \Phi_n} \right]^T \quad (26)$$

Taking the mentioned relation into consideration, it can be claimed that the following relation is governed: $c_k = -g(\Phi_k)$ (27)

In the steep descent method, p_k and c_k are equal and relations (26-27) are applied to obtain p_k . As mentioned before, the generated entropy is considered as the objective function in this study. The maximum efficiency achieved when the generated entropy is minimized [13,14].

10. TEMPERATURE AND VELOCITY BOUNDARY CONDITIONS

The fluid flow in the annulus pipe is assumed to be annular with $Re=1400$. The temperature of the inlet fluid is 300K while the walls are assigned to be at 400K. Having runned the computer program under the conditions mentioned above, temperature and velocity distribution are obtained by solving momentum and energy equations for the non-Newtonian fluid flow in annulus pipe. Running the computer program is repeated for different values of n and different geometries of fins, using Newton iteration method while the entropy generation is calculated at each step of all cases. Finally the optimum case with minimum entropy generation is developed for each value of n . the optimized fin characteristics are demonstrated in Table 1 for different values of n . it should be noted that $n=1$ associated with non-Newtonian fluid flow.

11. RESULTS OF THE COMPUTER PROGRAM

Figure 10, demonstrates velocity and temperature distribution in the non-Newtonian fluid flow in an annulus pipe equipped with some typical fins. The same problem is solved by the computer program, both for Newtonian and non-Newtonian flow and optimization results are tabulated in Table 1.

12. CONCLUSION

In the present study, a non-Newtonian fluid flowing in an annulus pipe has been simulated numerically by solving momentum and energy equations. Using second law of thermodynamics in the simulation procedure makes the entropy generation calculations possible, in the flowing domain. Some thermal fins are applied on the external frame of the internal pipe of the annulus, in order to improve heat transfer. The objective of the present study is to optimize the geometry of applied fins using entropy generation minimization method. Using EGM method provides the condition to simultaneously consider the effects of fin geometry on heat transfer and also pressure drop resulted by friction. The written code based on FEM, is runned under the conditions mentioned at sec.8 and for three different values of n . ($n=0.5, 1, 1.4$) the resulted optimum fin geometry is demonstrated in Table 1. It is worth noting that height and width of the optimum fin are approximately equal for all examined values of n . In the other word, whether the fluid is Newtonian or non-Newtonian, ($n=1$ or $n \neq 1$), the fin which results in maximum heat transfer conditions has equal width and height.

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